Best-Case Lower Bounds in a Group Sequence for the Job Shop Problem

Guillaume Pinot Nasser Mebarki

IRCCyN — UMR CNRS 6597
Nantes, France
firstname.lastname@irccyn.ec-nantes.fr

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Introduction

Group sequencing:

- is a scheduling method;
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A best-case evaluation of a group sequence could be interesting.

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Example: a Job Shop Problem

i: the index of the operations, $\Gamma^-(i)$: the set of the predecessors of O_i, m_i: the resource needed by O_i, p_i: the processing time needed by O_i.

A Job Shop Problem

i	1	2	3	4	5	6	7	8	9
$\Gamma^-(i)$	$\varnothing$	$\{1\}$	$\{2\}$	$\varnothing$	$\{4\}$	$\{5\}$	$\varnothing$	$\{7\}$	$\{8\}$
m_i	M_1	M_2	M_3	M_2	M_3	M_1	M_3	M_1	M_2
p_i	3	3	3	4	3	1	2	2	2

A Solution to This Problem

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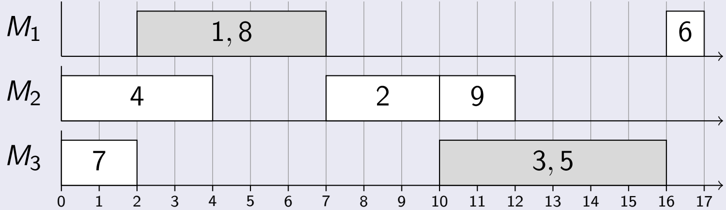
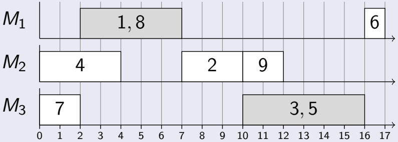
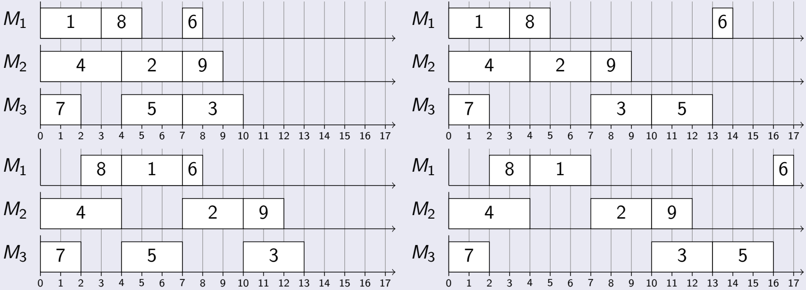


Execution of the Example

The Group Sequence



The Corresponding Semi-Active Schedules



Why is Group Sequencing Interesting?

Why is group sequencing interesting?

- predictive reactive method;
- flexibility on sequences;
- widely studied in the last twenty years: [Erschler and Roubellat, 1989, Wu et al., 1999, Artigues et al., 2005]
- no need to model the uncertainties;
- the method is able to absorb some uncertainties: [Wu et al., 1999, Esswein, 2003, Pinot et al., 2007];
- evaluation of the group sequence in the worst case in polynomial time for *minmax* regular objectives as $C_{\max}$ and $L_{\max}$.

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Algorithms

Intuitive Formulation

$$\begin{cases} \theta_i = \max\left(r_i, \max_{j \in g^-(i)} \chi_j, \max_{j \in \Gamma^-(i)} \chi_j\right) \\ \chi_i = \theta_i + p_i \end{cases}$$

Improved Formulation

$$\begin{cases} \theta_i = \max\left(r_i, \gamma_{g^-(i)}, \max_{j \in \Gamma^-(i)} \chi_j\right) \\ \chi_i = \theta_i + p_i \\ \gamma_{g_{\ell,k}} = C_{\max} \text{ of } 1|r_i|C_{\max}, \forall O_i \in g_{\ell,k}, r_i = \theta_i \end{cases}$$

- θ_i Lower bound of the starting time of O_i
- χ_i Lower bound of the completion time of O_i
- $\gamma_{g_{\ell,k}}$ Lower bound of the completion time of $g_{\ell,k}$

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Example

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i	$\Gamma^-(i)$	m_i	p_i	$g(i)$
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2	$\{1\}$	M_2	2	$g_{2,1}$
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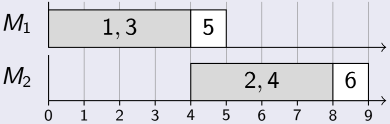

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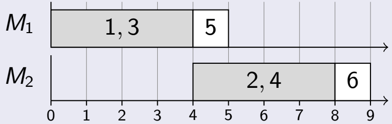

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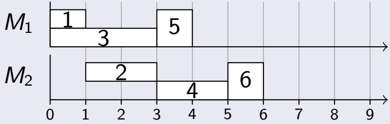

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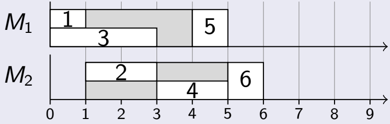

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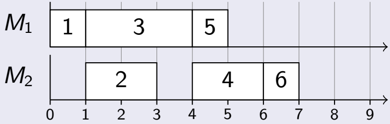

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Simple Lower Bound

It can be used directly to compute a lower bound of the group sequence:

$$\mathrm{LB}(L_{\max}) = \max_{\forall O_i} L_i(\chi_i) = \max_{\forall O_i}(\chi_i - d_i)$$

$$\mathrm{LB}(C_{\max}) = \max_{\forall g_{\ell,k}} \gamma_{g_{\ell,k}} \qquad \text{(Natural LB)}$$

Makespan Lower Bound

Classical job-shop lower bound: one-machine-problem relaxation [Carlier, 1982] on each machine.

The one-machine-problem relaxation require some tools:

- a head for each operations: θ_i;
- a tail for each operations: a reversed θ_i.

For group sequencing the relaxation is done on groups instead of machines (more subproblems, but smaller).

Solving the one-machine problems:

- using Jackson Preemptive Schedule: JPS OMP LB;
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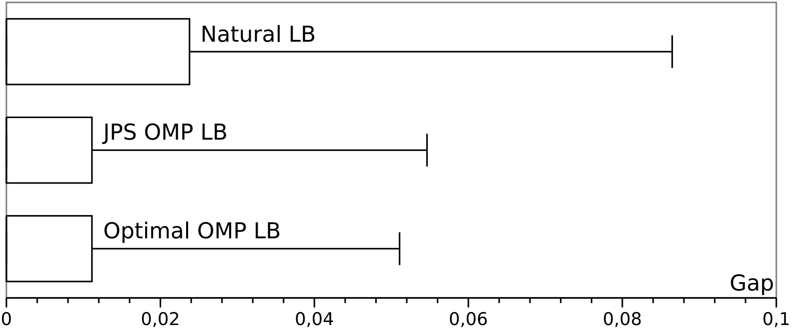
Gaps

Instances : `la01` to `la40` from [Lawrence, 1984].

For each instances, we generate a group sequence with

- a known optimal makespan[Brucker et al., 1994];
- a very high flexibility [Esswein, 2003].



Other results

Computation times:

$$\begin{aligned}\text{time(Optimal OMP LB)} &\simeq \text{time(JPS OMP LB)}\\ &\simeq 2 \times \text{time(Natural LB)}\end{aligned}$$

In an exact method using these lower bounds:

$$10 \times \text{time(exact(Optimal OMP LB))} \simeq \text{time(exact(JPS OMP LB))}$$

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Conclusion

We have proposed:

- different lower-bound tools;
- lower bounds.

They can be used directly:

- more complet description of a group sequence in its globality;
- its usage in a decision support system gives additional information to the operator.

These tools can also be usefull in:

- heuristics;
- exact methods.

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Thank You

Thank you for your attention.

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